STAGE-BY-STAGE MODELING OF THE THERMAL CONDITIONS OF COMPLEX SYSTEMS

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An approximate method for the analysis of the thermal conditions of complex objects is considered, consisting of the successive application of a series of mathematical models differing in the degree of detail of the temperature-field description.

In designing complex technical objects in instrument construction and mechanical engineering, it is often necessary to analyze their thermal conditions. The basic complexity of this analysis for many devices is associated with the difficulties in taking account of the mutual influence of the large bodies in thermal contact with energy sources and gas or liquid fluxes. A sufficiently complete mathematical description of the temperature field of such objects may be given in the form of a system of differential heat-conduction equations for solids

$$(c\rho)_i \frac{\partial T_i}{\partial \tau} = \operatorname{div}(\lambda_i \operatorname{grad} T_i) + q_{Vi}, \quad i = 1, \ldots, I \tag{1}$$

and energy equations for the heat-carrier fluxes

$$(c\rho)_l \left(\frac{\partial t_l}{\partial \tau} + \mathbf{v} \cdot \operatorname{grad} t_l \right) = \operatorname{div}(\lambda_l \operatorname{grad} t_l), \quad l = 1, \ldots, L \tag{2}$$

with boundary conditions of the first, second, and third kind or with matching conditions at the interfaces.

However, even with the use of modern computers, it is usually difficult or even practically impossible to realize such a complete mathematical model for complex objects. In addition, attempts to rigorously describe the temperature field in a complex system may prove inexpedient, not only because of the unwieldiness of the problem, but also because of our imprecise knowledge of the input information required for the calculation (heat-source powers, heat-transfer coefficients, thermophysical properties of the bodies, heat-carrier flow rates). At the same time, to ensure normal thermal conditions of a series of objects, it is required to calculate the temperature fields at different levels of the constructional hierarchy with different (often very high) degrees of detail. These difficulties are avoided by developing approximate methods of analysis of the heat-transfer processes in systems of bodies.

Experience in calculating the thermal conditions of complex objects in cryogenic electromechanical engineering [1], and optical and electronic instrument construction [2], shows the efficiency of the general approach to the analysis of temperature fields of a system of bodies known as the method of stage-by-stage modeling.

The essence of the given method is as follows. The thermal field of a complex system is calculated by the successive use of different thermal and mathematical models. A model describing the thermal conditions of the whole system with the minimal permissible degree of detail is first used. This stage allows the mean (e.g., volume-mean and surface-mean) temperatures of the bodies or groups of bodies and the mean flow temperatures of the heat-carrier fluxes to be determined. Then the system is broken down into its individual parts and more detailed analysis of their thermal conditions is performed. The heat-transfer conditions between each individual region of the system and the remaining components are described (i.e., the boundary conditions are specified) using information obtained in the preceding

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stages of the calculation. In the boundary conditions at the surfaces separating the part of the system considered in the given stage of the calculation, the mean values of the heat fluxes or temperatures of the surrounding bodies determined previously are used.

The construction of the models in the method of stage-by-stage modeling includes two parts: first, enlargement of the model for the calculation of certain mean characteristics; second, the separation of an element or group of elements of the system for detailed analysis.

There are two general methods of constructing an enlarged model. The first is to pass from a system of bodies to a quasihomogeneous body. This approach is possible in the case when the system consists of a sufficiently large number of constructionally similar elements, with repeating measurements (e.g., the heated zone of an electronic apparatus of cassette construction, the winding of an electrical machine). In the analysis of the heat conditions of such systems, they may be regarded as quasihomogeneous bodies with effective thermophysical properties. A model of the quasihomogeneous body allows the spatial distribution of the local mean (within the limits of the elementary cell) temperature [3]. The temperature distribution inside the elementary cell may later be analyzed in more detail.

In the other method of constructing the enlarged model, integral operations of averaging with respect to a certain coordinate or over the volume may be applied to the initial model. In this case, the problem either leads to equations describing a temperature field of smaller dimension or reduces to the problem of calculating the surface-mean and volume-mean temperatures.

Now consider the transition from a mathematical model describing the temperature field of a system of bodies using a system of partial differential equations to a model with point parameters. Within the framework of the latter model, the state of the elements of the system is characterized by one or a few values of the temperature, and the mathematical model takes the form of a system of algebraic (for steady problems) or ordinary differential (for nonsteady problems) equations.

To pass to a model with point parameters, the operator of averaging over the volume

$$I[f(\mathbf{x})] = \frac{1}{V_V}\int f(\mathbf{x})\,dV = f_V \tag{3}$$

is applied to Eqs. (1), (2). Then for the solid bodies we obtain

$$C_i \frac{dT_{iV}}{d\tau} = \int_{S_i} \lambda_i \operatorname{grad} T_i \cdot \mathbf{n} dS + P_i,\ i = 1, \ldots, I, \tag{4}$$

and for the heat-carrier fluxes

$$C_l \frac{dt_{lV}}{d\tau} = \int_{S_l'} \lambda_l \operatorname{grad} t_l \cdot \mathbf{n} dS - \int_{S_l'} c\rho_l \mathbf{v}\cdot\mathbf{n} t_l dS,\ l = 1, \ldots, L. \tag{5}$$

Taking into account that at the solid-body surfaces S_l' bounding the fluxes, the normal component of the velocity is zero ($\mathbf{v} \cdot \mathbf{n} = 0$), and assuming that conductive heat transfer through the surfaces $S_{l,\text{in}}$ and $S_{l,\text{out}}$ of the input and output cross sections of the flux may be neglected in comparison with the convective heat transfer, Eq. (5) is written in the form

$$C_l \frac{dt_{lV}}{d\tau} = \int_{S_l'} \lambda_l \operatorname{grad} t_l \cdot \mathbf{n} dS - \int_{S_{l,\text{in}}+S_{l,\text{out}}} c\rho_l \mathbf{v}\cdot\mathbf{n} t_l dS = \int_{S_l'} \lambda_l \operatorname{grad} t_l \cdot \mathbf{n} dS - c_l (G_{\text{out}} t_{l,\text{out}} - G_{\text{in}} t_{l,\text{in}}). \tag{6}$$

Equations (4), (6) express, in integral form, the energy-conservation law for the i-th body and the l-th flux. The next step of the transition to a model with point parameters is to specify the expressions for the heat flux passing through the surfaces S_i and S_l'. In many cases it proves possible to describe the heat transfer between the bodies and between the bodies and the fluxes using the thermal conductivities. The thermal conductivities σ_{i-j} and σ_{i-l} are now introduced; by definition,

$$\sigma_{i-j} = P_{i-j}/(T_{Si,n} - T_{jS,m}), \tag{7}$$

$$\sigma_{i-l} = P_{i-l}/(T_{iS,n} - t_{lV}). \tag{8}$$

Substituting the expressions for the heat fluxes at the boundaries into Eqs. (4) and (6), the result is a system of ordinary differential equations in terms of the mean temperatures

$$C_i \frac{dT_{iV}}{d\tau} = \sum_{n=1}^{N_i} \left[\sum_{j=1}^{I} \sigma_{i-j} (T_{jS,m} - T_{iS,n}) + \sum_{l=1}^{L} \sigma_{i-l} (t_{lV} - T_{iS,n}) \right] + P_i, \; i = 1, \ldots, I, \tag{9}$$

$$C_l \frac{dt_{lV}}{d\tau} = \sum_{i=1}^{I} \sigma_{i-l} (T_{iS,n} - t_{lV}) - c_l (G_{\text{out}} t_{l,\text{out}} - G_{\text{in}} t_{l,\text{in}}), \; l = 1, \ldots, L. \tag{10}$$

The following unknowns appear in Eqs. (9), (10): for the solid bodies, the volume-mean temperature T_{iV} and the mean temperature $T_{iS,n}$ of the sections into which the surface is divided in order to specify the heat fluxes to the surrounding bodies and cavities; for the heat carriers, the volume-mean temperature t_{lV} and the mean flow temperature of the outgoing flow $t_{l,\text{out}}$. The temperature $t_{l,\text{in}}$ is either specified, if the flux enters the system from outside, or calculated from the temperature of the outgoing fluxes whose mixing forms the given flux.

The number of unknown temperatures in the system (9), (10) exceeds the number of equations (I + L). Therefore, closure of the system entails the use of additional equations establishing a relation between the desired temperatures.

Some methods of closing the system of equations are now considered. The simplest method is to assume that the temperature field in the bodies is uniform; this is correct if the internal thermal resistance is much less than the external value (Bi << 1):

$$T_{iV} = T_{iS,1} = \ldots = T_{iS,n}. \tag{11}$$

In some systems, further enlargement of the model is possible: some bodies are assumed to be equal in temperature, groups of bodies are combined, and a model of the system with a smaller number of bodies is then considered. The basis of this approach and algorithms of model enlargement were given in [4, 5].

Sometimes, knowledge of the surface-mean temperature T_{iS} of a given body is sufficient in order to describe its heat transfer with its surroundings. In this case closure of the system of equations is secured by introducing the coefficient of nonuniformity of the temperature field

$$\Psi_i = \frac{T_{iS} - \tilde{T}_i}{T_{iV} - \tilde{T}_i}, \quad \tilde{T}_i = \sum_j \sigma_{i-j} T_{jS} / \sum_j \sigma_{i-j}, \tag{12}$$

where $\tilde{T}_i$ is the temperature of an arbitrary medium for the i-th body.

For bodies with internal heat sources, in steady conditions, Ψ_i may be approximately specified by means of the well-known solution for canonical bodies [3]

$$\Psi = \left(1 + \frac{1}{n} \text{Bi}\right)^{-1}, \tag{13}$$

where n = 3, Bi = $\alpha l/\lambda$ for a plate of thickness 2l; n = 4, Bi = $\alpha R/\lambda$ for an infinite cylinder; and n = 5 for a sphere.

In nonsteady conditions, the generalized dependence for regular conditions may be used [3]:

$$\Psi = (H^2 + 1.44\,H + 1)^{-1}, \; H = \frac{\alpha K S}{\lambda V}, \tag{14}$$

where H is the generalized Biot number; K is the shape factor of the body.

In establishing the relation between the flux temperatures t_{lV}, $t_{l,\text{in}}$, and $t_{l,\text{out}}$, two approaches are most often used:

1) ideal mixing of the heat carrier in the cavity is assumed, so that

$$t_{l,\,\mathrm{out}} = t_{l,V}, \tag{15}$$

2) it is assumed that

$$t_{lV} = (t_{l,\,\mathrm{out}} + t_{l,\mathrm{in}})/2. \tag{16}$$

The nonuniformity coefficient of the liquid-flux temperature field may be introduced

$$\Psi_l = (t_{lV} - t_{l,\mathrm{in}})/(t_{l,\,\mathrm{out}} - t_{l,\mathrm{in}}), \tag{17}$$

which is estimated from a consideration of some model problems. For example, for a one-dimensional flow in a channel of length h with a wet perimeter U at constant wall temperature and with a local heat-transfer coefficient α:

$$\Psi_l = \left[1 - \exp\left(-\frac{\alpha U h}{Gc}\right)\right]^{-1} - \frac{Gc}{\alpha U h}. \tag{18}$$

The problem of closing the system of equations considered for the example of transition to a model with point parameters arises in any enlargement of the model by means of averaging of the initial equations. The meaning of closure of the enlarged model consists in the approximate recovery of information on the temperature fields lost in performing the averaging. The adequacy of the enlarged model and its applicability for the calculation of mean temperatures and heat fluxes (under the condition that the initial model is adequate) are determined by the correctness of the methods of closure adopted.

After constructing an enlarged model, the next step is to separate the system into parts and analyze the individual regions. According to the basic idea of the method, the values of the mean temperatures and heat fluxes obtained in the preceding stage of the calculation using the enlarged model are used in specifying the boundary conditions for the region of the system considered in this stage. The temperature distribution $T(\mathbf{x})$ or heat-flux density $q(\mathbf{x})$ at the boundaries of the region or the temperature distribution of the surrounding ideal medium $T_c(\mathbf{x})$ are replaced by their mean values $\langle T_n \rangle$, $\langle q_n \rangle$, $\langle T_{cn} \rangle$ over the sections Γ_n:

$$\langle f_n \rangle = \frac{1}{S_n} \int_{S_n} f(\mathbf{x})|_{\Gamma_n}\, dS, \quad f = T,\ q,\ T_c.$$

The possibility of replacing the spatial distribution of the thermal perturbations at the boundaries by mean perturbations is based on the principle of local influence, according to which "any local perturbation of the temperature field is local and does not propagate to remote sections of the field" [3]. Thanks to the action of the local-influence principle, less-detailed description of the heat-transfer processes in the regions of the system remote from those that are currently of interest is possible. Deviation of the true parameter values from the mean level is regarded as a local perturbation.

The quantitative aspect of the local-influence principle was investigated in [6] for problems of replacing complex spatial distributions of energy sources by simpler forms. Investigation of the error in calculating the temperature fields due to perturbations of the boundary conditions is necessary in order to establish the basis for the method of stage-by-stage modeling. The solution of this problem forms the subject of our next work, in which this error is estimated.

NOTATION

T_i, t_l, temperature of i-th body and l-th flux; λ, thermal conductivity; $c\rho$, volume specific heat; τ, time; α, heat-transfer coefficient; q_V, specific power of heat sources; P, total power; C, total specific heat; V, volume; S, surface area; G, mass flow rate; P_{i-j}, σ_{i-j}, heat flux and conduction between bodies i and j; Ψ, nonuniformity coefficient. Subscripts: V, volume mean; S, surface mean; S, n, surface mean over the n-th section; in, out, values at the cavity inlet and outlet.

LITERATURE CITED

1. I. A. Glebov, G. N. Dul'nev, A. Yu. Potyagailo, and A. V. Sigalov, "Mathematical modeling of the thermal conditions of a cryoturbogenerator rotor," Izv. Akad. Nauk SSSR, Energ. Trans., No. 2, 71-77 (1982).

2. G. N. Dul'nev, B. V. Pol'shchikov, and A. Yu. Potyagailo, "Development of an algorithm of nonhierarchical modeling of heat-transfer processes in complex radioelectronic complexes," Radiotekhnika, 34, No. 11, 49-54 (1979).
3. G. N. Dul'nev and É. M. Semyashkin, Heat Transfer in Radioelectronic Apparatus [in Russian], Énergiya, Leningrad (1968).
4. G. N. Dul'nev and A. Yu. Potyagailo, "Classification of thermal models of systems of many bodies with energy sources," Tr. LITMO, No. 86, 4-13 (1976).
5. G. N. Dul'nev, "Local-influence principle in heat-conduction theory," Tr. LITMO, No. 37, 17-24 (1959).